```
clear; close all; clc
set(0,'DefaultFigureWindowStyle','docked')
```

Monte Carlo integration

Approximate the area (volume) of a region without analitically integrating it

Area of circle (π)

First start with a well known shape, the circle

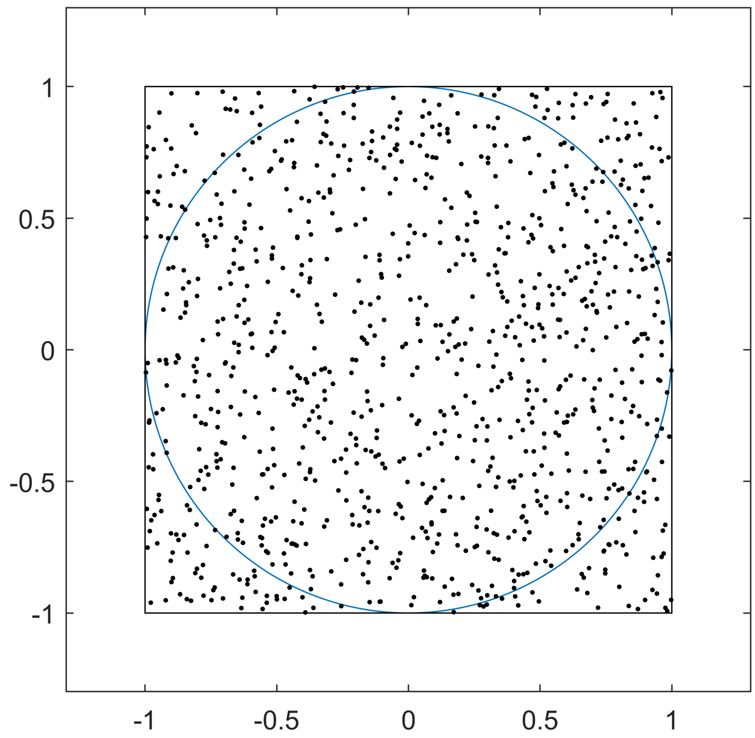
For visualizing the probleme let us draw a circle with a bouinding square

```
% Radius
R = 1;
% Angular variable
t = 0:0.01:2*pi;
x = R * cos(t);
y = R * sin(t);

figure(1)
hold on; box on; axis equal
plot(x,y)
rectangle('Position',[-1,-1,2,2])
xlim([-R,R]*1.3)
ylim([-R,R]*1.3)
```

Throw random points onto this square

```
n = 1e3;
%p = zeros(2,n);
p = rand(2,n) * 2 * R - R;
plot(p(1,:),p(2,:),'k.')
```



Count the points that lie in the circle

```
c = 0;
for i = 1:n
    % Check if vector is longer than the radius
    if sqrt(p(1,i)^2+p(2,i)^2) < R
        c = c + 1;
    end
end
c
```

```
c = 782
```

```
% Faster solution of counting
% Root sum of squares: that is length of a vector: rssq(p)
% The indices of the points that are inside the circle rssq(p)<R
c = sum(rssq(p)<R)
```

```
c = 782
```

```

% The area is the square's area times the fraction of points inside the circle
area = 4*R^2 * c/n
```

```
area = 3.1280
```

```
% Realtive error
```

```
area / (R^2*pi) - 1
```

```
ans = -0.0043
```

Area under a function (integration)

Monte Carlo sampling

Approximate a function with particles. Particles are weighted samples taken from the function.

Let the function be $f(x) = \mathrm{sinc}(x)$

$$\mathrm{sinc}(x) = \begin{cases} \dfrac{\sin(x)}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

Plot the function

```
% Range
xmin = -5;
xmax = 5;
x = xmin : 0.01 : xmax;
y = sinc(x);
figure(2)
plot(x,y,'b')
hold on
```

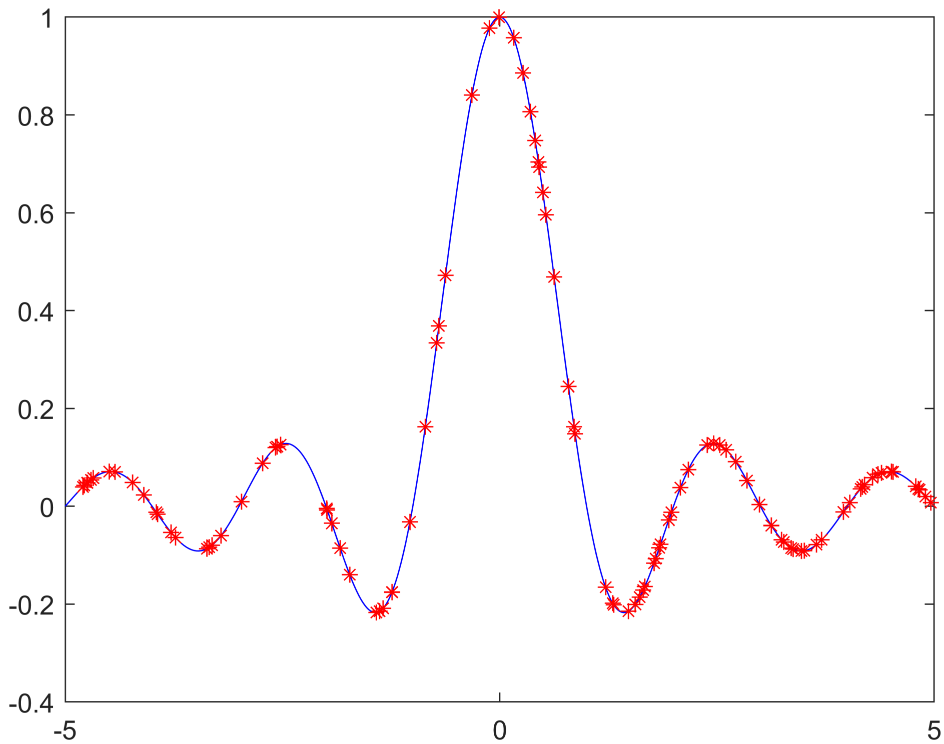
Approximate this function with particles

Sample the function in the interval that we are interested in

```
% Number of samples
n = 100;
% Sampling points
s = rand(1,n) * (xmax - xmin) + xmin;
% The samples
z = sinc(s)
```

```
z = 1×100
   -0.0158    0.0685    0.1250    0.9775    0.4722   -0.0783    0.0333   -0.0535 ···
```

```
% Plot the samples
plot(s,z,'r*')
```



Sampling from a proposal function

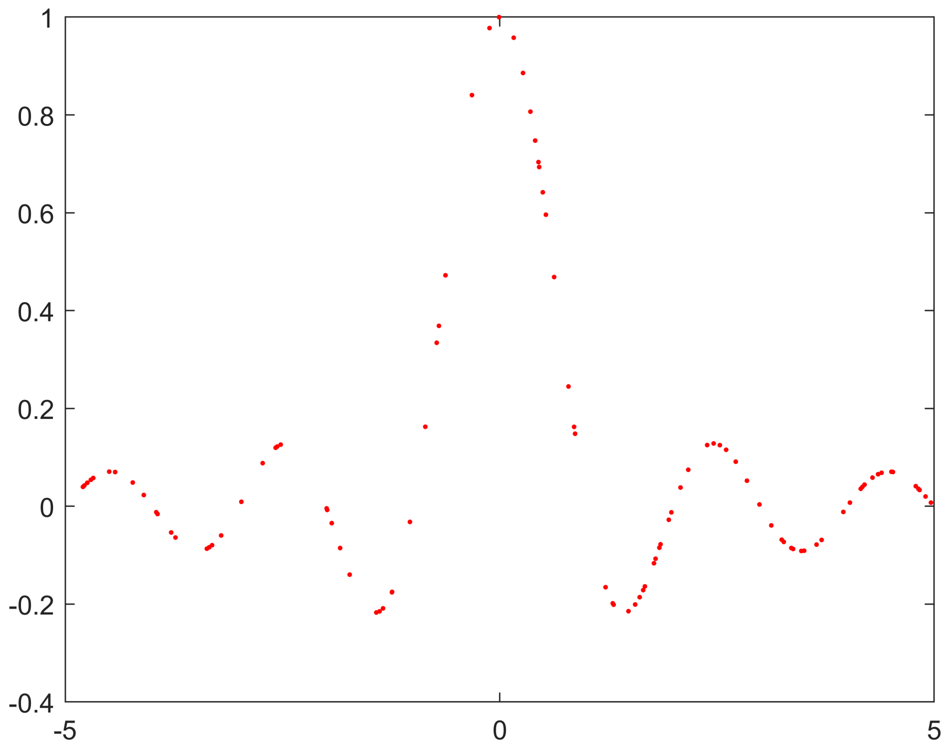
What if we cannot sample from $f(x)$ but only from $g(x)$?

In that case we have to weight the samples by the factor $w(x) = \dfrac{f(x)}{g(x)}$

Let $g(x) = \sin(x)$

$$w(x) = \frac{\operatorname{sinc}(x)}{\sin(x)} = \frac{1}{x}$$

```
z1 = sin(pi*s);
w = 1./(pi*s);
z2 = z1 .*  w;
figure(3)
plot(s,z2,'r.')
```

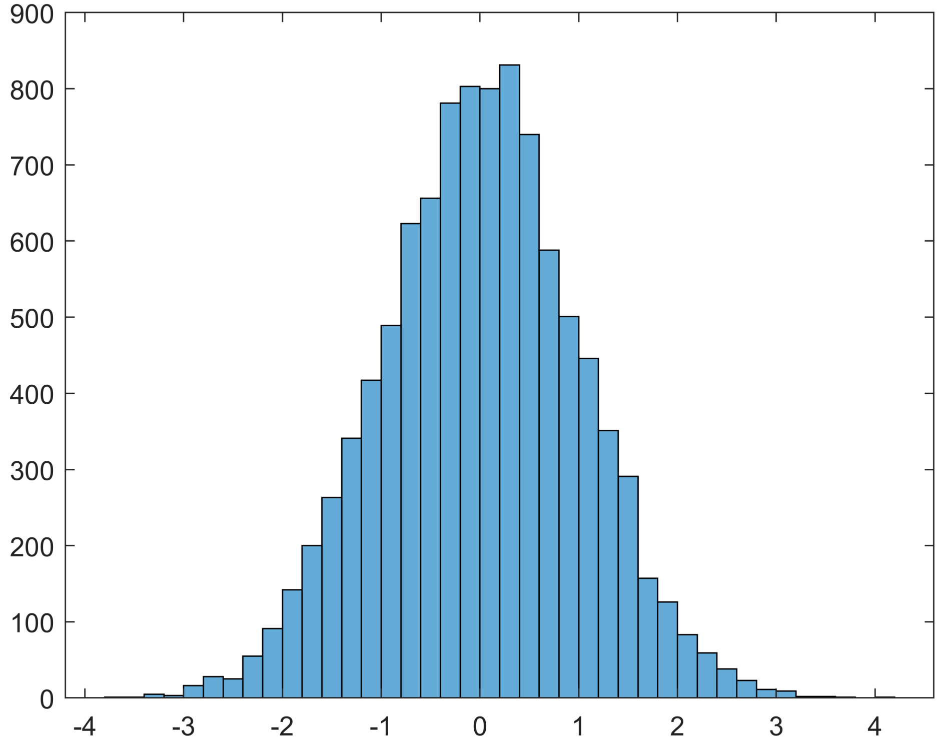


Approximate a PDF with particles

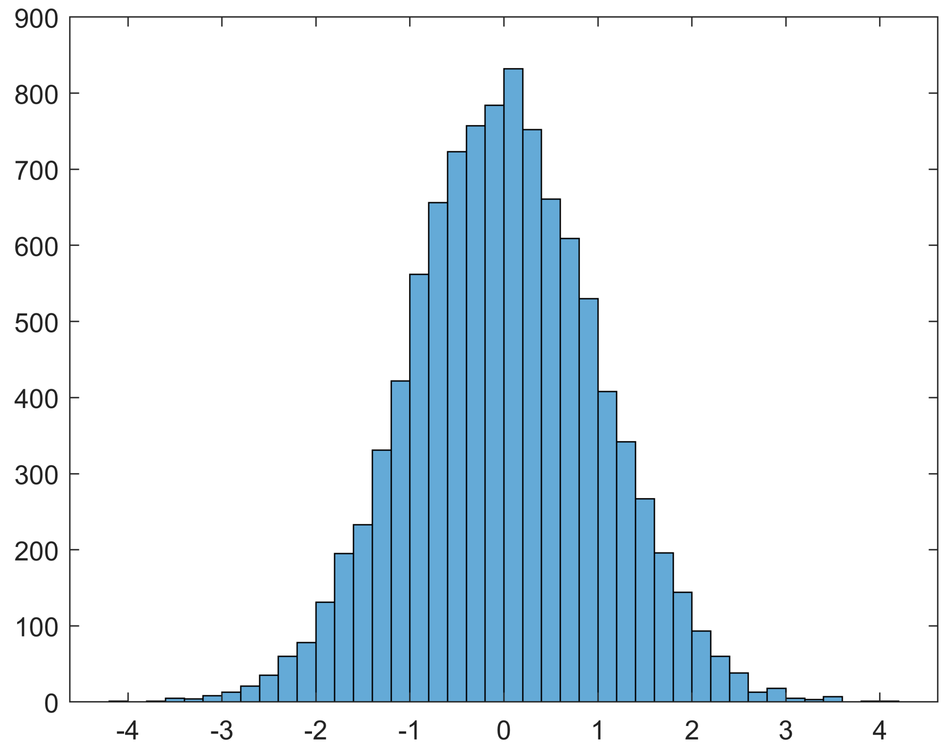
We need to sample more particles where the PDF is higher. But how?

We should uniformly sample the 'y' axis and transform the values to the 'x' axis. This means we need to inverte the function.

```
n = 1e4;
x = -1 + 2*rand(1,n);
y = sqrt(2) * erfinv(x);
figure
histogram(y)
```




```
y = sqrt(-2*log(rand(1,n))).*cos(2*pi*rand(1,n)); % or sin
figure
histogram(y)
```




```
plot(y,normpdf(y),'k.')
```